

The investigation of the Orientation of Galaxies in Structures

Elena Panko ¹, Paulina Piwowska ², Jolanta Godłowska ³,
Włodzimierz Godłowski ⁴, Piotr Flin ⁵

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1. Kalinenkov Astronomical Observatory, Nikolaev National University, Nikolskaya, 24, 54030 Nikolaev, Ukraine email: panko.elena@gmail.com

2. Uniwersytet Opolski, Institute of Physics, ul. Oleska 48, 45-052 Opole, Poland e-mail: paoletta@interia.pl

3. Department of Monitoring and Modelling Air Pollution, Institute of Meteorology and Water Management, 30-215 Krakow, Borowego 14, Poland email: Jolanta.Godlowska@imgw.pl

4. Uniwersytet Opolski, Institute of Physics, ul. Oleska 48, 45-052 Opole, Poland e-mail: godlowski@uni.opole.pl

5. Institute of Physics, Jan Kochanowski University, 25-406 Kielce, Swietokrzyska 15, Poland email: sfflin@cyf-kr.edu.pl

Abstract

The investigation of the orientation of galaxies is a standard test concerning to scenarios of galaxy formation, because different theories of galaxy formation make various predictions regarding to the angular momentum of galaxies. The

new method of analysis of the alignment of galaxies in clusters was proposed in the paper [Godłowski (2012)] and now is improved. We analyzed the distribution of the position angles of the galaxy major axes, as well as the distribution of two angles describing the spatial orientation of galaxy plane, which gives the information about galaxy angular momenta. We discuss the orientation of galaxies in groups and clusters of galaxies. The results show the dependence of the alignment with respect to clusters richness. The implications of the results for theories of galaxy formation are discussed as well.

keywords angular momenta, galaxies

1 Introduction

Different theories of galaxy formation (for example [Peebles (1969), Zeldovich (1970), Sunyaew & Zeldovich (1972), Doroshkevich (1973), Shandarin(1974), Dekel (1985), Wesson (1982), Silk & Efstathiou (1983), Bower et al. (2006)]) make different predictions regarding to the angular momentum of galaxies. These classical theories of formation of galaxies and its structures were then revised and improved by many researchers (for example [Lee & Pen (2000), Lee & Pen (2001), Lee & Pen (2002), Navarro (2004), Mo et al. (2005), Brook et al. (2008), Vera-Ciro et al. (2011), Paz et al. (2011)), Shandarin et al.(2012), Giahi-Saravani & Schfer (2013), Trujillo et al. (2006), Codis et al. (2012), Giahi-Saravani & Schfer (2013)]) but this statement remains valid. More generally, the observed variations in angular momentum represent simple but fundamental constraints for any model of galaxy formation [Romanowsky & Fall (2012)].

In the commonly accepted Λ CDM model, the Universe seems to be spatially flat, as well as homogeneous and isotropic at the same appropriate scale. In this model the structures were formed from the primordial adiabatic, nearly scale invariant Gaussian random fluctuations ([Silk (1968), Peebles & Yu (1970), Sunyaew & Zeldovich (1970)]). This picture is in agreement with both the nu-

merous numerical simulations [Springel et al. (2005), van de Weygaert & Bond (2008a), van de Weygaert & Bond (2008a), Codis et al. (2012)] and the observations. Unfortunately the angular momenta of galaxies are known only for very few galaxies. Therefore instead of the angular momenta, the orientation of galaxies is investigated. In order to this either the orientation of position angles of major galaxy axis [Hawley & Peebles (1975)], or spatial orientation of galaxy planes [Oepik (1970), Jaaniste & Saar(1978), Flin & Godłowski (1986)] is investigated. [Godłowski et al. (2005)] suggest that the alignment should increase with richness of the cluster.

It also should be noticed that it is commonly agree that for groups and clusters of galaxies there is no evidence of rotation and groups and clusters of galaxies do not rotate (for example [Regos & Geller (1989), Diaferio & Geller (1997), Diaferio (1999), Rines et al. (2003)] - see however [Kalinkov et al. (2005)] for opposite opinion). Moreover recently [Hwang & Lee (2007)] examined dispersions and velocity gradient of 899 Abell clusters and found a possible rotation in only six of them. Thus, any non-zero angular momentum of groups and clusters of galaxies would just come from possible alignment of galaxy spins.

The idea that richer clusters we can expect more strong alignments is results both from analysis of implications of theoretical relations between the angular momentum and the mass of the structure and from analysis of the observational results of alignment of galaxies in different scales. It is of course true that presently it is commonly believed that galaxies are formed before clusters but our idea is not in conflict with this. It can be explained both in the tidal-torque scenario [Heavens & Peacock (1988), Catelan & Theuns (1996), Fedeli (2013)] and in the Li model [Li (1998)].

The prediction that the alignment should increase with richness of the cluster was confirmed by [Aryal (2007)] but both [Godłowski et al. (2005)] and [Aryal (2007)] analysis was qualitative only. Therefore this problem was analyzed in details in later papers [Godłowski et al. (2010), Godłowski (2011a), Godłowski (2011b),

Godłowski (2012)]. In this works it was found that degree of the alignment of galaxies orientations in clusters depend on theirs number of members and increase with the amount of the galaxies members. It is equivalent to the existence of a relation between anisotropy and the number of galaxies in a cluster. Moreover it was found that orientations of galaxies analyzed sample of rich Abell galaxy clusters are not random, i.e., that there exists an alignment of galaxies in rich Abell galaxy clusters. In the present paper we would like to investigated this problem deeper, improving new method of analysis of the alignment of galaxies in clusters proposed in the paper [Godłowski (2012)].

Table 1: The results of numerical simulations for positions angles P , Tullys' groups.

Test	\bar{x}	$\sigma(x)$	$\sigma(\bar{x})$	$\sigma(\sigma(x))$
χ^2	16.9524	1.4592	0.0461	0.0326
$\Delta_1/\sigma(\Delta_1)$	1.2513	0.1543	0.0048	0.0034
$\Delta/\sigma(\Delta)$	1.8772	0.1581	0.0050	0.0035
C	-1.0256	0.9295	0.0294	0.0208
λ	0.7317	0.0615	0.0019	0.0014
$\Delta_{11}/\sigma(\Delta_{11})$	0.0065	0.2346	0.0074	0.0052

Table 2: The statistics of the observed distributions for real Tullys' groups.

	P		δ_D		η	
Test	\bar{x}	$\sigma(x)$	\bar{x}	$\sigma(x)$	\bar{x}	$\sigma(x)$
χ^2	16.800	1.152	17.983	1.601	17.267	1.579
$\Delta_1/\sigma(\Delta_1)$	1.218	0.176	1.356	0.116	1.509	0.208
$\Delta/\sigma(\Delta)$	2.081	0.218	2.040	0.172	2.343	0.205
C	0.026	0.932	0.044	0.992	1.268	0.984
λ	0.834	0.072	0.740	0.055	1.829	0.064
$\Delta_{11}/\sigma(\Delta_{11})$	-0.283	0.182	0.292	0.237	0.773	0.267

2 Overview

The orientation of galaxies is usually investigated either by analysing the distribution of the position angle of the galactic major axes or by the analysis of the spatial orientation of the normal line to the galaxy main plane in the investigated coordinate system. To obtain the spatial orientation of galaxies for each galaxy, two angles are determined: δ_D - the angle between the normal to the galaxy plane and the main plane of the coordinate system, and η - the angle between the projection of this normal onto the main plane and the direction towards to the zero initial meridian.

In our previous papers weve shown that analysis of the spatial orientation of galaxies planes can be used as a general, standard test of galaxies forming scenario [Flin & Godłowski (1986), Godłowski (1993), Godłowski (1994), Godłowski & Ostrowski (1999), Godłowski et al. (2010), Flin et al. (2011), Godłowski (2011a)]. Any study of galactic orientation based on the projection of galaxies on the celestial sphere gives a four-fold ambiguity in the solution for angular momentum. By the reason of none information connected with the direction of the galaxy spin our analysis is reduced to only two solutions. Using the Supergalactic coordinate system ([Flin & Godłowski (1986)] based on [Sandage & Tammann (1976)]) the following relations between angles (L, B, P) and (δ_D, η) hold:

$$\sin \delta_D = -\cos i \sin B \pm \sin i \cos r \cos B, \quad (1)$$

$$\sin \eta = (\cos \delta_D)^{-1} [-\cos i \cos B \sin L + \sin i (\mp \cos r \sin B \sin L \pm \sin r \cos L)], \quad (2)$$

$$\cos \eta = (\cos \delta_D)^{-1} [-\cos i \cos B \cos L + \sin i (\mp \cos r \sin B \cos L \mp \sin r \sin L)], \quad (3)$$

where $r = P - \pi/2$. As a result of the reduction of our analysis into two solutions it is necessary to consider the sign of the expression: $S = -\cos i \cos B \mp \sin i \cos r \sin B$ and for $S \geq 0$ reverse sign of δ_D respectively [Godłowski et al. (2010)].

Significant progress in the investigation of galaxy plane orientation was made

Table 3: The results of numerical simulation - sample of 247 cluster each with number of members galaxies the same as in the real cluster.

	P		δ_D		η	
Test	\bar{x}	$\sigma(x)$	\bar{x}	$\sigma(x)$	\bar{x}	$\sigma(x)$
χ^2	34.9798	0.5364	35.5824	0.5461	36.3663	0.5332
$\Delta_1/\sigma(\Delta_1)$	1.2550	0.0419	1.2523	0.0428	1.3756	0.0473
$\Delta/\sigma(\Delta)$	1.8788	0.0436	1.8844	0.0492	2.0208	0.0518
C	-1.0195	0.3749	-0.5226	0.3807	-0.1667	0.4043
λ	0.7720	0.0168	0.8099	0.0201	0.8193	0.0188
$\Delta_{11}/\sigma(\Delta_{11})$	0.0014	0.0645	0.0039	0.0151	0.0083	0.0705

Table 4: The statistics of the distributions for position angles for sample of 247 Abell clusters.

	Equatorial coordinates		Supergalactic coordinates	
Test	\bar{x}	$\sigma(\bar{x})$	\bar{x}	$\sigma(\bar{x})$
χ^2	36.8591	0.5924	36.7899	0.6315
$\Delta_1/\sigma(\Delta_1)$	1.7046	0.0622	1.7021	0.0626
$\Delta/\sigma(\Delta)$	2.2663	0.0594	2.2746	0.0591
C	1.1940	0.4530	1.1220	0.4237
λ	0.9177	0.0240	0.9138	0.0220
$\Delta_{11}/\sigma(\Delta_{11})$	-0.0005	0.0855	0.0940	0.0924

by [Hawley & Peebles (1975)]. They discussed in detailed manner the method of investigating the galaxies orientation through analysing distribution of position angles as well as the influence of possible errors and observational effects. [Hawley & Peebles (1975)] analysed the distributions of position angles using the χ^2 -test, Fourier test and the autocorrelation test. Since [Hawley & Peebles (1975)] this method was accepted as standard method for investigation of a galactic alignment. There are several modification of the original [Hawley & Peebles (1975)] method [Flin & Godłowski (1986), Kindl(1987), Godłowski (1993), Godłowski (1994), Godłowski & Ostrowski (1999), Aryal & Saurer (2000), Godłowski et al. (2010), Godłowski (2012)]

In order to detect non-random effects in the distribution of the investigated angles: δ_D , η and P we divided the entire range of the analyzed angles into n bins and carried out three different statistical tests. These tests were: the χ^2 test, the autocorrelation test and the Fourier test [Hawley & Peebles (1975), Godłowski et al. (2010), Godłowski (2011b), Godłowski (2012)]. We analyzed two samples of galaxy clusters. We computed the value of analyzed statistics for each cluster and later the mean value of the analyzed statistics. It was compared with theoretical predictions and results of numerical simulations obtained with using RANLUX generator [Luescher (1994), Luescher (2010)]. We used 6 statistics discussed in detail in [Godłowski (2012)].

At first we analyzed the sample of 18 Tullys' clusters (see also [Godłowski (2011b)]). The results are presented in the Tables 1 and 2. One should note, that there are insignificant differences with results in Table 2 and results in [Godłowski (2011b)] (Table 5) because of a small differences in method of computing the inclination angle with taking into account "true" axial ratio q_0 which depends on morphological type and converting axial ratio q to standard photometrical axial ratio (see [Fouque & Paturel (1985)]). From our results it is clearly seen that for samples of 18 groups of galaxies we do not found any significant alignments.

The next step was the analysis of the distribution of the position angles in the

sample of rich Abell clusters [Godłowski et al. (2010), Godłowski (2012)]. This sample was taken from [Panko & Flin(2006)] catalog, which is catalog of galaxy cluster obtain on the base of the Muenster Red Sky Survey [Ungruhe et al. (2003)]. Our sample contains 247 Abell clusters with richness at least 100 galaxies each being identified with one of ACO clusters [Abell et al. (1989)]. The results of our investigation are presented in the Tables 3 and 4. Now, on the contrary to the previous investigated samples of Tully groups, we found significant deviation of mean values of the statistics from expected values obtained from numerical simulations. From the Figure 1 and Table 3 we could find that results of the expected value of analyzed statistics for angles δ_D and η are larger than that obtained for position angles p . It is mostly caused of the fact that during the process of deprojection of the spatial orientation of galaxies from its optical images we obtain two possible orientations - see equations 1-3. From analysis of these equations it is easy to see that both solutions are not independent and as a result the distribution of analyzed statistics is modified and must be obtained from numerical simulations. Such differences have not changed our conclusion obtained from analysis of Tullys' groups of galaxies, because we have not found significant deviation even if we took smaller values obtained from simulations of position angles, but it should be taken into account during future analysis of the spatial orientation of galaxies in our sample of rich Abell clusters.

3 Conclusions

In the present paper we confirmed the predictions from the previous analysis [Godłowski et al. (2010), Godłowski (2011b), Godłowski (2012)] which showed the dependency of the alignment of galaxies in clusters on richness of the cluster. Moreover, we found that for the sample of Tullys' groups of galaxies we do not found any significant alignment while it is observed for the sample of 247 Abells' clusters, with richness at least 100 galaxies each. In the present paper

we extended our analysis by showing that expected values of analyzed statistics for angles giving spatial orientation of galaxies, δ_D and η are greater than that obtained for position angles of major axis of galaxies. It is mostly because during the analysis of the spatial orientation of galaxies we have two possible orientations of galaxies which are not independent to each other. Our results lead to the conclusion that the angular momentum of the cluster increases with the mass of the structure. With such a dependency it is natural to expect that in rich clusters significant alignment should be present. Usually a dependency between the angular momentum and the mass of the structure is presented as empirical relation $J \sim M^{5/3}$ [Wesson (1979), Carrasco (1982), White (1984), Brosche (1986), Paz et al.(2008)), Romanowsky & Fall (2012)], for review see also [Schaefer (2009)]. In our opinion the observed relation between the richness of the galaxy cluster and the alignment is due to tidal torque, as suggested by [Heavens & Peacock (1988)] and [Catelan & Theuns (1996)]. Moreover, the analysis of the linear tidal torque theory is pointing in the same direction [Noh & Lee (2006a), Noh & Lee (2006b)]. They've noticed the connection of the alignment with the considered scale of the structure. It should be also noticed that many of other result connected with galaxy alignment is interpreted in the light of tidal torque theory. For example Gonzalez and Teodoro [Gonzalez-Sanchez, & Teodoro (2010)] interpreted the alignment of just the brightest galaxies within a cluster as an effect of action of gravitational tidal forces. However one should note that our result, increase of alignment with cluster richness is also compatible with the prediction of the Li model in which galaxies are formed in the rotating universe ([Li (1998), Godłowski et al. (2003), Godłowski et al. (2005)]).

We should keep in mind that Li's model [Li (1998)] remains valid only provided rotation occurs on a sufficiently large scale. Thus in this model considerations concerning angular momenta of galaxy structures will be also valid in the case of large scale, but not necessarily global, rotation of the Universe. It

is important because observed the amplitude of quadrupole fluctuations value provided by both COBE and WMAP measurements is only $\Delta T^2 = 249 \mu K^2$ [Hinshaw et al., (2007)]. More generally the large scale anisotropy linked with the rotation of the Universe, homogeneous magnetic field, anisotropy of curvature and other similar factors is strongly restricted by WMAP observations of quadrupole anisotropy of relic radiation (see for review Demianski & Doroshkevich [Demianski & Doroshkevich]). One should note however that most serious problem of Lis model [Li (1998)] is consists in the fact that the observed amount of rotation of spiral galaxies cannot arise from the Universes rotation alone, since the required amount of rotation of the Universe on the order obtained earlier by Birch [Birch (1982), Birch (1983)] which is too large in comparison with the detected anisotropies of cosmic background radiation (for review see [Godłowski (2011a)])

Recently, there have been also some attempts to investigate galaxy angular momenta on a large scale. [Paz et al.(2008))] analysing galaxies from the Sloan Digital Sky Survey catalogue found that the galaxy angular momenta are aligned perpendicularly to the planes of large-scale structures, while there is no such effect for the low-mass structures. They interpret this as consistent with their simulations based on the mechanism of tidal interactions. The change of alignment with the surrounding neighbourhood was observed also in alignment study in void vicinity [Varela et al. (2012)] being continuation of earlier study of galaxy orientation in regions surrounding bubble-like voids [Trujillo et al. (2006)]. [Jones et al.(2010)] found that the spins of spiral galaxies located within cosmic web filaments tend to be aligned along the larger axis of the filament, which they interpreted as "fossil" evidence indicating that the action of large scale tidal torques effected the alignments of galaxies located in cosmic filaments. [Tempel et al. (2013)] found evidence that the spin axes of bright spiral galaxies have a weak tendency to be aligned parallel to filaments. For elliptical/S0 galaxies, they have a statistically significant result that the spin axes of ellipticals are aligned preferentially

perpendicular to the host filaments. Lee [Lee (2011)] comparing of his observational results with the analytic model based on the tidal torque theory reveals that the spin correlation function for the late-type spiral galaxies follow the quadratic scaling of the linear density correlation and that the intrinsic correlations of the galaxy spin axes are stronger than that of the underlying dark halos. The intrinsic correlations between galaxy spins and intermediate principal axes of the tidal shears was also found by Lee and Erdogdu [Lee & Erdogdu (2007)].

Possible relation between filament and orientation of galaxies was noticed also by Godowski & Flin [Godłowski & Flin(2010)]. In this paper the orientation of galaxy groups in the Local Supercluster was studied, and it was found a strong alignment of the major axis of the groups with directions towards the supercluster center (Virgo cluster) as well as with the line joining the two brightest galaxies in the group. The Interpretation of these observational results was following. The brightest galaxies of the group, believed to be the most massive ones, originated first. Afterward, the hierarchical clustering leads to aggregation of galaxies around these two galaxies. The groups are formed on the same or similarly oriented filaments. It should be also noticed that galaxy cluster intrinsic alignments to very large scales of $100h^{-1}\text{Mpc}$, representing a tendency of clusters to point preferentially towards other clusters was found by Smargon et al.[Smargon et al. (2012)] .

Possible significance of the evolution of the alignment with redshift is suggested by the results of the paper of [Song & Lee (2012)], which found that the alignment profile of cluster galaxies drops faster at higher redshifts and on smaller mass scales. Moreover, one should note that the largest scale alignment was found in the series of paper by Hutsemekers ([Hutsemekers (1998), Hutsemekers & Lamy (2001), Hutsemekers et al. (2005)]) during analysis of the alignment of quasar polarization vectors. The polarization vectors appear to be coherently oriented or aligned over huge (about 1 Gpc) regions at the sky. Furthermore, the mean polarization angle θ appears to rotate with redshift at the

rate of about 300 per Gpc. These results usually are not questioned, (with exception of Joshi et al. [Joshi et al. (2007)] which not found any effects during analysis of theirs sample [Jackson et al. (2007)]), however the origin of this effect is still discussed. While interpretations like a global rotation of the Universe can potentially explain the effect the properties they observed qualitatively correspond to the dichroism and birefringence predicted by the photon-pseudoscalar oscillation within a magnetic field. The possible interpretations was discussed many times for example by Hutsemekers [Hutsemekers & Lamy (2001), Hutsemekers et al. (2005), Hutsemekers et al. (2010)] and recently by Agarwal, Kamal and Jain [Agarwal (2011)]

In our further paper we would like to extend our consideration to more detailed analysis of the distribution of two angles δ_D and η , describing the spatial orientation of the galaxy plane. Moreover, we would like to investigate does the effect found in the present paper depends on the cluster BM type and velocity dispersion of member galaxies.

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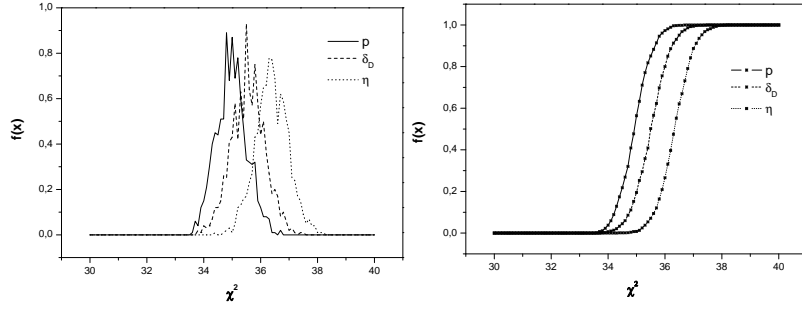


Figure 1: Differences in Probability Distribution Function (PDF) (left panel) and Cumulative Distribution Function (CDF) (right panel), for angles P , δ_d and η (χ^2 statistics). The figure was obtained from 1000 simulations of sample of 247 clusters each with number of member galaxies the same as in the real cluster.